Active noise control without secondary path modelling – Fx-LMS with step size sign and value updated using the artificial intelligence

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Summary
The Filtered-x LMS algorithm, when applied to active noise control, uses a secondary path model in order to update control filter parameters. The model is, however, subject to an error if the plant changes, what can result in algorithm divergence. Additional excitation signal can be then used to update the model on-line. Active control system properties are also dependant on the algorithm step size value. Its inappropriate choice impairs convergence rate on one hand, and on the other hand can make the algorithm divergent. In the literature, there are some algorithms that require no secondary path modelling. They are of heuristic or theoretic origin. This paper focuses on developing an alternative solution. It proposes a modification of the Fx-LMS, which updates sign and value of the step size using artificial intelligence to provide stable operation and noise reduction. This modification does not require modelling the acousto-electric plant and updating the model during control system operation, if properties of the plant change. The proposed algorithm is then validated by simulation experiments based on real-plant data.

1. Introduction
Active Noise Control (ANC) is usually explained by destructive interference of the unwanted primary noise with secondary sound generated by a control unit. One of the most common algorithms for ANC systems is Filtered-reference LMS, commonly known as FXLMS. The structure of this algorithm is presented in Figure 1. In order to generate the phase-inverted sound at the desired area, the algorithm should take into account the acousto-electric path, as defined from the control filter output \( u \) to the sampled and conditioned error microphone signal \( e \). Such path is referred to as the secondary path, \( S \). In Figure 1, \( P \) is the primary acousto-electric path defined between the reference \( x \) and the error microphone \( e \) signals. The paths include electronic components used for filtering, sampling, reconstructing (applies to the secondary path), and conditioning those signals. Index \( i \) refers to the discrete time sample. The FXLMS algorithm in feedforward structure uses the reference signal filtered by a secondary path model \( \hat{S} \), which in Figure 1 is noted as \( r \).

Remaining signals, \( d \) and \( y \) are the primary and secondary signals at the desired area of noise reduction. The feedforward control filter \( (W) \) is updated according to [7]:
\[
\mathbf{w}(i + 1) = \mathbf{w}(i) - \mu e(i) r(i),
\]
where
\[
\mathbf{w}(i) = [w_0 \ w_1 \cdots w_{N-1}]^T,
\]
\[
\mu = \text{adaptation step-size},
\]
\[
r(i) = [r(i), r(i - 1), \cdots r(i - N + 1)]^T,
\]
\[
x(i) = [r(i), r(i - 1), \cdots r(i - M + 1)]^T.
\]
In (1), \( N \) is the order of the control filter, while \( M \) is the order of the secondary path FIR model with parameters collected in vector \( \hat{S} = [\hat{s}_0, \hat{s}_1, \cdots, \hat{s}_{M-1}] \).

Figure 1. Filtered-x LMS structure.
The Fx-LMS algorithm converges as long as the phase and amplitude estimation errors satisfy the following condition [9,10]:

\[ \mu(i) = \min_f \frac{2\cos(\phi_f)}{A_f |S_f|^2 |x_f(i)|^2}, \]  

(2)

for every signal component frequency \( f \), where \( \phi_f \) is the phase estimation error, and \( A_f \) is the amplitude estimation error.

The process of secondary path modelling can be performed offline if the plant is time-invariant or changes little. When a plant variation during the system operation can make phase modelling error higher than 90°, online identification is frequently proposed to keep the convergence. Otherwise, the system diverges [12]. However, the online identification requires usually an additional excitation signal, which deteriorates performance of the ANC system.

In the literature, there are some propositions of model-free algorithms and modifications to well-known algorithms used for noise control [2, 3, 5, 6, 9, 10]. Pawelczyk in [6] proposed to simplify Fx-LMS, by using approximated delay of the plant in order to guarantee acceptable phase error. For wideband signals he recommended to process them in subbands and apply sampling frequency conversion for better tuning. Then, Zhou and DeBrunner in [10] used heuristic parameter search to find the adaptation direction, which would make the algorithm to converge. This idea was later expanded by Wu et.al. in [9], taking into account more directions to increase the convergence rate. Chen and Chang in [3] used an adaptive evolutionary optimisation directly to adapt the control filters weights. A fuzzy inference system was constructed by Kurczyk and Pawelczyk in [5] in order to update the sign of the algorithm step size, without a need of calculating fitness functions. In turn, George and Panda in [11] prosed using a particle-swarm optimisation based approach.

2. Changing sign and tuning value of the algorithm step size

As it was presented by Zhou and DeBrunner in [10], the convergence of the FxLMS algorithm can be analysed on the complex-plane. Assuming that the modelling error is represented as:

\[ \hat{S}_f = A_f S_f e^{j\phi_f}, \]

(3)

and the reference signal is a tonal excitation of given frequency \( f \), the parameters of the adaptive filter \( W \) can be represented as in Figure 2.

During the adaptation process, the control filter tends towards the optimal solution \(-P/S_f\) (algorithm converges), or runs away from that solution (algorithm diverges). In a situation, when the algorithm diverges, a phase correction is required to adapt the algorithm in a proper direction. This is achieved in [10] by simply switching the sign of adaptation step size in (1). When the modelling error is close to 90° then changing the sign of the step size does not help. Kurczyk and Pawelczyk considered in [5], additional phase correction by delaying the reference signal. The benefit of that approach is that the delay does not need to follow from modelling the secondary path. Its role is to change the adaptation direction, in any side. Even one sample delay suffices. Together with update of the sign of the step size, it guarantees convergence. However, if the delay is chosen more carefully, and the adaptation direction is as close as possible to the direction pointing to the optimal solution, it may significantly increase convergence rate of the algorithm, as proposed by the authors in [4]. The convergence can also be substantially supported by appropriate choice of the step size value. In this paper, a memetic algorithm is used to adapt both the sign and value of the adaptation step size. The modelling error compensation by applying the time-variant delay instead of the secondary path model is not addressed in this paper, but it can be combined with this algorithm directly using the approach from [4].

2.1. Application of a memetic algorithm

The structure of the algorithm presented in this paper is shown in Figure 3.
Figure 3. Model-free ANC system, with sign and value of the step size adapted by a memetic algorithm.

Figure 4. Memetic algorithm applied for updating LMS step-size.

A. Initialize
During the initialize step, the population of $P_{size}$ chromosomes is generated. Each chromosome contains an array of $n$-bits (0 or 1). Each array codes a value of the adaptation step size. Information of the step size sign is not included in the chromosome, but later during ‘local optimisation’. All chromosomes are generated randomly.

B. Fitness Evaluation
At this stage, the algorithm lets the adaptation to be carried out for each chromosome in active population. Each chromosome is being tested for $T_{fit}$ samples at maximum. The test finishes earlier if the residual error signal energy reaches its limit defined as $E_{abort}$. If that happens, the local search is invoked.

C. Selection
Selection creates a new population out of the old one. The mechanism used for selecting new parents is the roulette wheel parent selection. For each chromosome in active population there is a probability of being selected for reproduction step, given as:

$$P(m_i) = \frac{fit(m_i)}{\sum_{j=1}^{P_{size}} fit(j)}$$

(4)

where $P(m_i)$ is a probability of being selected for reproduction; $m$ is a member index.

D. Crossover
In this step for each pair of parents there is a probability $cprob$ for creating two children out of two parents. Crossover combines the information of two parental chromosomes using a method called single-point crossover and dividing parental chromosomes into four parts. Then part A of the first parent is combined with part B of the second parent. The other child chromosome is created out of remaining parts. If crossover is not invoked upon two parents, then the children are identical as their parents.

E. Mutation
Mutation is performed for each child with $mprob$ probability. If mutation occurs, mutated chromosome negates one of his bits, at random.

F. Local search
During the local search new fitness function values are evaluated. Only the chromosomes that reached their $E_{abort}$ limit are tested, but with a switched sign. The sign for each chromosome is
stored separately form the coded value of the step size. Performing ‘local search’ only for those values of the step size, where system diverges, minimises the number of necessary evaluations.

G. Stop conditions
For this algorithm there are two different and independent stop conditions. The first one is activated, when the full cycle of the genetic optimization repeats for \( \text{Epoch} \) times. The second condition is activated when:

\[
\Delta E_e(i) > E_{stop},
\]

(5)

where:

\[
\Delta E_e(i) = E_e(i_m)/E_e(i),
\]

(6)

and \( i_m \) is the sample number for each chromosome in the population, when the fitness evaluation activates for that chromosome, \( E_e \) is the energy estimator, calculated as:

\[
E_e(i) = 0.99 \cdot E_e(i-1) + 0.01 \cdot E_e(i).
\]

(7)

H. Optimisation criterion
The fitness function for the proposed algorithm is given as:

\[
fit(\mu_m(i)) = 1/E_e(i).
\]

(8)

3. Simulation analysis
All simulations conducted in this paper are based on data recorded in a power plant. The paths are simulated as FIR filters, with 250 parameters each. Phase and magnitude responses of the plant are given in Figure 5. This figure is split into coloured groups. In reference to the amplitude response it is suspected that the system can successfully operate and has a satisfactory convergence rate in the green area. For frequencies marked in red the electronics used suppresses the signal. Anyway the genetic algorithm can try to scale the adaptation step size to its maximum value to let the control algorithm operate and improve convergence rate. As for the phase response, the classic LMS algorithm should converge in the white areas. They represent all frequencies, where the phase modelling error is lower than \( 90^\circ \). In the violet areas the phase modelling error exceeds \( 90^\circ \), and, therefore, a switch of step size sign should be forced.

3.1. Response to phase modelling error time variation
The simulation conducted in this paper assumes that the plant changes in time, in order to test the potential of the proposed modification to Fx-LMS. The change is forced by additional 2-sample delay introduced to the plant, after 5s. For the first simulation, a tonal noise of 504 Hz is considered first. For this frequency the phase of the secondary path is \( \varphi_1(504) = 145^\circ \), (when

![Figure 5. Amplitude (to the top) and phase (to the bottom) responses of the identified plant.](image-url)
wrapped as in Figure 5, and after introducing the delay it is \( \varphi_2(504) = -32^\circ \). Parameters of the memetic algorithm are as follows:

\[
P_{\text{size}} = 10; \quad \text{Epoch} = 20; \quad n = 4; \quad cprob = 0.6; \quad mprob = 0.05; \quad T_{\text{init}} = 200; \quad E_{\text{abort}} = 1.5; \quad E_{\text{stop}} = 2.
\]

Simulation results are presented in Figures 6 and 7. The modified algorithm converges easily with noise reduction level of 26 dB, although it does not use any information about the secondary path. Next, a narrowband noise of frequencies from 430 to 550 Hz is considered. The control and inference systems use the same set of parameters as shown above. Obtained results are presented in Figures 8 and 9. Before 5\(^{th}\) sec, phase estimation error was \( \varphi_f \in (90^\circ, 270^\circ) \), for each \( f \) component. In that situation it is necessary to change the sign of the adaptation step size in order to guarantee algorithm convergence. After 5\(^{th}\) s, the wrapped phase error \( \varphi_f \in (-90^\circ, 90^\circ) \). The algorithm responds properly to the change of the secondary path. Noise reduction level of 15 dB is achieved.

### 4. Conclusions

In this paper a modified memetic algorithm has been proposed to adapt the sign and value of the step size of the LMS algorithm. The update procedure does not require secondary path modelling.
Theoretical expectations have been validated by simulation experiments using data from a real power plant.

Proposed algorithm converges even for a small population size. However, its convergence cannot be guaranteed then due to stochastic nature of evolutionary algorithms. Increasing the population size and number of epochs results in more desired results.

Simulation experiments have demonstrated that the algorithm operates satisfactorily for a tonal and narrowband noises. However, it can easily be extended to wider noise by using a subband architecture as in [9] at the expense of computational complexity.

The proposed method is an alternative to classic Fx-LMS algorithm, and it is particularly useful for ANC systems where the secondary path is subject to change.

Acknowledgement

The research reported in this paper has been supported by the National Science Centre, decision no. DEC-2012/07/B/ST7/01408, and by the Ministry of Science and Higher Education, Poland.

References